An Anisotropic Vector Play Model Using Decomposed Shape Functions

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This paper proposes a new anisotropic vector play model using the decomposition of vector shape functions. The parallel and perpendicular components of magnetic field *H* to the magnetic flux density *B* are independently identified to construct decomposed shape functions. This paper further proposes a method to reconstruct the perpendicular component from the one-dimensional (1-D) measurement of parallel component based on a magnetic energy approach.

***Index terms*—Anisotropy, magnetic energy, play model, vector hysteresis.**

I. Introduction

The vector play model [1]-[3] is one of the most accurate and efficient models to represent the vector hysteretic property of silicon steel sheets. Using the isotropic vector play model and the anisotropy matrix, [3] successfully approximated the anisotropic vector hysteresis of non-oriented silicon steel sheets including the elliptic rotational field. However, because of the simple construction of anisotropy matrix, the representation may not be highly accurate when the anisotropy becomes strong.

For the accurate anisotropy representation, this paper proposes anisotropic shape functions that represent the perpendicular component of the magnetic field $\boldsymbol{H}$ to the magnetic flux density $\boldsymbol{B}$ independently of its parallel component. Based on a magnetic energy approach [4], [5], this paper further proposes a method to reconstruct the perpendicular component without its direct measurement only using the 1-D measurement of the parallel component.

II. Vector Play Model

An anisotropic vector play model provides an output of magnetic field $\boldsymbol{H}$ from an input of magnetic flux density $\boldsymbol{B}$:

$$\boldsymbol{H} = \boldsymbol{P}(\boldsymbol{B}) = \int_0^{B_S} \boldsymbol{f}\left(\zeta, \boldsymbol{p}_\zeta(\boldsymbol{B}), \theta_B\right) \mathrm{d}\zeta \tag{1}$$

$$\boldsymbol{p}_\zeta(\boldsymbol{B}) = \boldsymbol{B} - \frac{\zeta\left(\boldsymbol{B} - \boldsymbol{p}_\zeta^0\right)}{\max\left(\zeta, \left|\boldsymbol{B} - \boldsymbol{p}_\zeta^0\right|\right)} \tag{2}$$

where $\boldsymbol{f}$ is an anisotropic vector shape function, $\boldsymbol{p}_\zeta$ is the vector play hysteron having radius ζ, $\boldsymbol{p}_\zeta^0$ is the vector $\boldsymbol{p}_\zeta$ at the previous time-point, θ_B is the azimuth angle of $\boldsymbol{B}$, and B_S is the saturation magnetic flux density. Ref. [3] proposed a simple form of shape function as

$$\boldsymbol{f}(\zeta, \boldsymbol{p}_\zeta(\boldsymbol{B}), \theta_B) = \boldsymbol{W}(\boldsymbol{B})\, \boldsymbol{f}_0(\zeta, \boldsymbol{p}_\zeta(\boldsymbol{B})) \tag{3}$$

where $\boldsymbol{W}$ is the anisotropy matrix and $\boldsymbol{f}_0$ is the isotropic vector shape function.

This paper proposes a more accurate representation of shape function as

$$\begin{aligned}\boldsymbol{f}\left(\zeta, \boldsymbol{p}_\zeta(\boldsymbol{B}), \theta_B\right) = &\ f_\parallel\left(\zeta, \left|\boldsymbol{p}_\zeta(\boldsymbol{B})\right|, \theta_B\right)\boldsymbol{e}_{p_\parallel} \\ &+ f_\perp\left(\zeta, \left|\boldsymbol{p}_\zeta(\boldsymbol{B})\right|, \theta_B\right)\boldsymbol{e}_{p_\perp}\end{aligned} \tag{4}$$

where $f_\parallel$ and $f_\perp$ are the shape functions for the parallel and perpendicular component of $\boldsymbol{H}$ to $\boldsymbol{p}_\zeta$, $\boldsymbol{e}_{p\parallel}$ and $\boldsymbol{e}_{p\perp}$ denote the parallel and perpendicular unit vector to $\boldsymbol{p}_\zeta$. The shape functions $f_\parallel$ and $f_\perp$ are determined from the measured parallel and perpendicular components of $\boldsymbol{H} = (H_\parallel(B, \theta_B), H_\perp(B, \theta_B))$ using the identification method for a scalar play model [6].

III. Magnetic Energy Approach

A. Magnetic energy and 2-D magnetic property

The vector $(H_\parallel(B, \theta_B), H_\perp(B, \theta_B))$ are measured under alternating flux along the θ_B-direction. The measurement of $H_\perp(B)$ requires the 2-D or rotational single sheet tester (SST) [7] whereas $H_\parallel(B, \theta_B)$ can be measured by the conventional 1-D SST or the Epstein frame using strip samples cut along the various directions.

Refs. [4] and [5] proposed a useful method to reconstruct $H_\perp(B, \theta_B)$ without its measurement using the magnetic energy. The magnetic energy F is defined as

$$F(B, \theta_B) = \int_0^{\boldsymbol{B}} \boldsymbol{H} \cdot d\boldsymbol{B} = \int_0^{B} H_\parallel\left(B, \theta_B\right) \mathrm{d}B. \tag{5}$$

It is assumed that $\boldsymbol{H}$ and $\boldsymbol{B}$ have no hysteretic relation and that the integration is independent of the path. Using the magnetic energy (5), the components of $\boldsymbol{H}$ are given as

$$H_\parallel = \frac{\partial F}{\partial B}, \quad H_\perp = \frac{1}{B}\frac{\partial F}{\partial \theta_B}. \tag{6}$$

Using the nonhysteretic expression of (5) and (6), this paper develops the determination method of $f_\perp$, where two magnetization curves without hysteresis, normal magnetization curve and anhysteretic curve, are used for the derivation.

B. Derivation from normal magnetization curve

The normal magnetization curve $H_m(B_m, \theta_B)$ is obtained by the measurement of symmetric B-H loops under alternating magnetic flux, where B_m is the amplitude. The play model does not distinguish the normal magnetization curve from the initial magnetization curve and express $\boldsymbol{H}_\mathrm{m} = (H_{m\parallel}, H_{m\perp})$ as

$$H_{m\parallel}(B_m, \theta_B) = \int_0^{B_m} f_\parallel(\zeta, B_m - \zeta, \theta_B)\mathrm{d}\zeta \tag{7}$$

$$H_{m\perp}(B_m, \theta_B) = \int_0^{B_m} f_\perp\left(\zeta, B_m - \zeta, \theta_B\right)\mathrm{d}\zeta. \tag{8}$$

From (5) and (7) the magnetic energy F is represented as

$$F(B_m,\theta_B)=\int_0^{B_m}\left\{\int_0^{B_S} f_{\parallel}(\zeta,p_\zeta,\theta_B)\mathrm{d}\zeta\right\}\mathrm{d}B$$
$$=\int_0^{B_m}\left\{\int_\zeta^{B_m} f_{\parallel}(\zeta,B-\zeta,\theta_B)\mathrm{d}B\right\}\mathrm{d}\zeta \tag{9}$$

From (6) and (9), $H_{m\perp}$ is given as

$$H_{m\perp}(B_m,\theta_B)=\int_0^{B_m}\frac{1}{B_m}\frac{\partial}{\partial\theta_B}\left\{\int_\zeta^{B_m} f_{\parallel}(\zeta,B-\zeta,\theta_B)\mathrm{d}B\right\}\mathrm{d}\zeta \tag{10}$$

From the comparison of (8) with (10), $f_\perp$ is estimated as

$$f_\perp(\zeta,B_m-\zeta,\theta_B)=\frac{1}{B_m}\frac{\partial}{\partial\theta_B}\left\{\int_\zeta^{B_m} f_{\parallel}(\zeta,B-\zeta,\theta_B)\mathrm{d}B\right\}. \tag{11}$$

Equation (11) is rewritten as

$$f_\perp(\zeta,B,\theta_B)=\frac{1}{B+\zeta}\frac{\partial}{\partial\theta_B}\left\{\int_0^{B} f_{\parallel}(\zeta,B,\theta_B)\mathrm{d}B\right\}. \tag{12}$$

C. Derivation from anhysteretic curve

Anhysteretic curve $\boldsymbol{H}_b(B_b, \theta_B)$ is obtained by the decaying alternating magnetic flux with biased direct magnetic flux B_b. The play model expresses $\boldsymbol{H}_b(B_b, \theta_B)$ as

$$H_{b\parallel}(B_b,\theta_B)=\int_0^{B_S} f_{\parallel}(\zeta,B_b,\theta_B)\mathrm{d}\zeta \tag{13}$$

$$H_{b\perp}(B_b,\theta_B)=\int_0^{B_S} f_{\perp}(\zeta,B_b,\theta_B)\mathrm{d}\zeta\ . \tag{14}$$

From (5) and (13), F is represented as

$$F(B_m,\theta_B)=\int_0^{B_S}\left\{\int_0^{B_b} f_{\parallel}(\zeta,B,\theta_B)\mathrm{d}B\right\}\mathrm{d}\zeta\ . \tag{15}$$

From (6) and (15), $H_{b\perp}$ is given as

$$H_{b\perp}(B_b,\theta_B)=\int_0^{B_S}\frac{1}{B_b}\frac{\partial}{\partial\theta_B}\left\{\int_0^{B_b} f_{\parallel}(\zeta,B,\theta_B)\mathrm{d}B\right\}\mathrm{d}\zeta\ . \tag{16}$$

From the comparison of (14) with (16), $f_\perp$ is estimated as

$$f_\perp(\zeta,B,\theta_B)=\frac{1}{B}\frac{\partial}{\partial\theta_B}\left\{\int_0^{B} f_{\parallel}(\zeta,B,\theta_B)\mathrm{d}B\right\}. \tag{17}$$

IV. Simulation Results

This section compares simulation results of the anisotropic vector models with measured magnetic property of non-oriented silicon steel sheet JIS: 35A230 under the alternating magnetic flux condition.

Fig.1 shows simulated B-$H_\perp$ loops using the shape functions (3) and (4) for alternating flux with $\theta_B = \pi/4$[rad], where $f_\perp$ in (4) is identified from measured $H_\perp$. Fig.2 shows simulated B-$H_\perp$ loops using the shape function $f_\perp$ determined from (12) and (17) without using measured B-$H_\perp$ loops.

Simulated B-$H_\parallel$ loops and hysteresis loss will be shown in the full paper, which agree with measured data.

The simulated B-$H_\perp$ loops given by the proposed vector model using $f_\perp$ agrees with measured loops. The shape function $f_\perp$ identified from measured $H_\perp$ gives B-$H_\perp$ loops very accurately. The shape function $f_\perp$ determined from (17) obtains more accurate B-$H_\perp$ loops than $f_\perp$ determined from (12). Fig.2 implies that the 2-D anisotropic vector hysteretic property can be reconstructed from the B-H loops measured with the 1-D SST, using the proposed anisotropic vector play model and the magnetic energy equation. The representation of rotational hysteretic property will be discussed in the full paper.

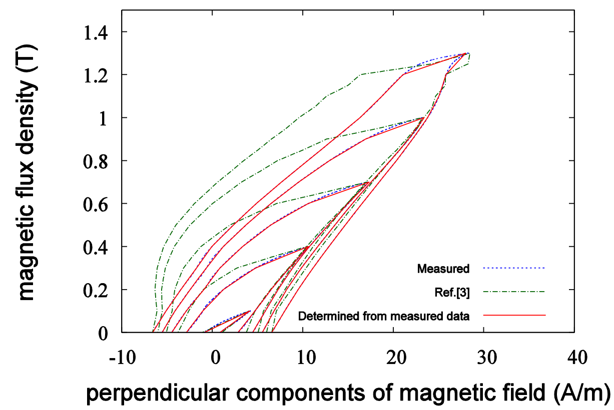

Fig. 1. Simulated B-$H_\perp$ loops using the shape functions (3) and (4), where $f_\perp$ in (4) is identified from measured data.

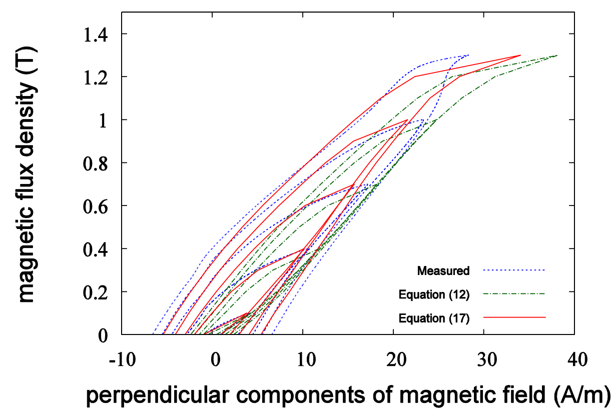

Fig. 2. Simulated B-$H_\perp$ loops using the shape function $f_\perp$ determined from magnetic energy by (12) and (17).

V. References

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